Newton-Raphsons metode

Maple udregninger og plots

af John V. Petersen

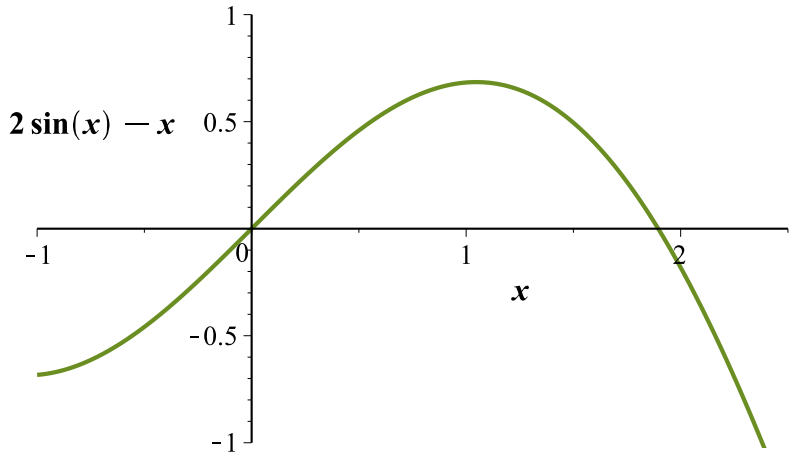
Indhold:

eks 1 i artiklen: a. Newton-Raphsons metode
Find en tilnærmet værdi for nulpunktet for funktionen $f(x) = 2 \cdot \sin x - x$, for $x \in]0, \pi[$.

```
> f := x→2·sin(x) − x;
```

$$f := x \to 2 \sin(x) - x \qquad \textbf{(1)}$$

```
> with(plots) :
> plot(f(x), x = -1..2.5, view = [-1..2.5, -1..1], scaling = constrained, thickness = 2, color
      = "OliveDrab", labels = [x, f(x)], labelfont = [helvetica, bold, 12]);
```



Newton iterationen

$$> NewtonStep := x - \frac{f(x)}{diff(f(x), x)} ;$$

$$NewtonStep := x - \frac{2\sin(x) - x}{2\cos(x) - 1} \quad \textbf{(2)}$$

Antal betydende cifre

```
> Digits := 5;
```

$$Digits := 5 \quad \textbf{(3)}$$

Startgættet ($\frac{\pi}{2}$ med 5 betydende cifre)

```
> x0 := 1.5708 ;
```

$$x_0 := 1.5708 \quad \textbf{(4)}$$

```
> x1 := eval(NewtonStep, x = x0 );
```

$$x_1 := 2.0000 \quad \textbf{(5)}$$

```
> x2 := eval(NewtonStep, x = x1 );
```

$$x_2 := 1.9010 \quad \textbf{(6)}$$

```
> x3 := eval(NewtonStep, x = x2 );
```

$$x_3 := 1.8955 \quad \textbf{(7)}$$

```
> x4 := eval(NewtonStep, x = x3 );
```

$$x_4 := 1.8955 \quad \textbf{(8)}$$

```
> x5 := eval(NewtonStep, x = x4 ) :
> x6 := eval(NewtonStep, x = x5 ) :
>
```

Newton iteration plots

> $graf := plot(f(x), x = -1..2.5, view = [-1..2.5, -1..1], scaling = constrained, thickness = 3, color = \text{"OliveDrab"}, labels = [x, f(x)], labelfont = [helvetica, bold, 12])$:

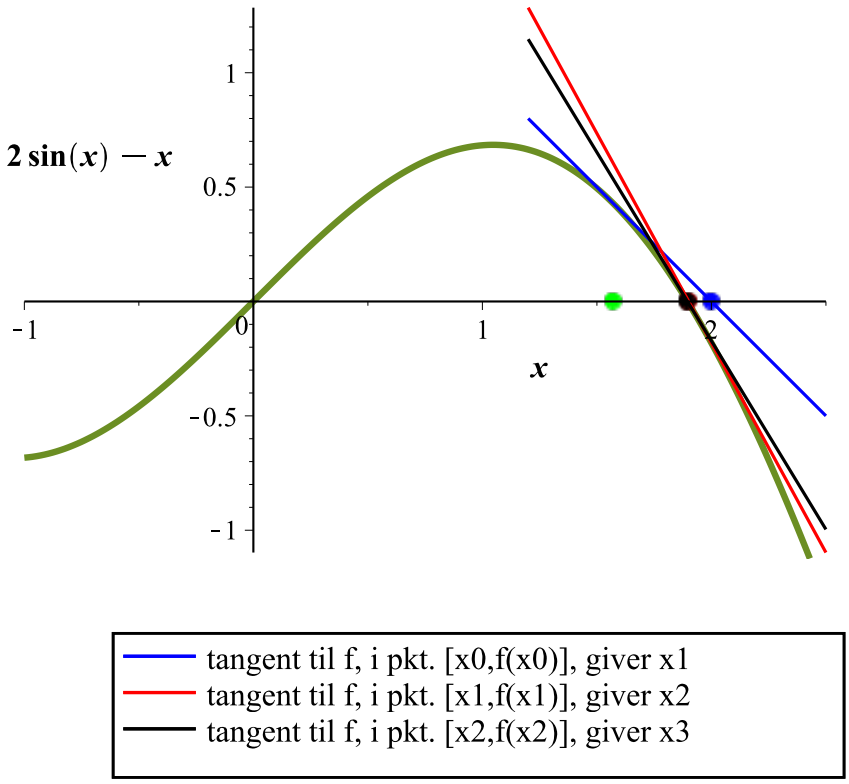
> $punkter := pointplot\left(\left[\left[\frac{\pi}{2}, 0\right]\right], color = green, symbol = solidcircle, symbolsize = 16\right), pointplot([2, 0], color = blue, symbol = solidcircle, symbolsize = 16), pointplot([1.9010, 0], color = red, symbol = solidcircle, symbolsize = 16), pointplot([1.8955, 0], color = black, symbol = solidcircle, symbolsize = 16)$:

> $tangentx0 := plot\left((2\cdot\cos(x_0) - 1)\cdot x + 2\cdot(\sin(x_0) - \cos(x_0)\cdot x_0), x = 1.2..2.5, color = blue, legend = \text{"tangent til f, i pkt. [x0,f(x0)], giver x1"}\right)$:

> $tangentx1 := plot\left((2\cdot\cos(x_1) - 1)\cdot x + 2\cdot(\sin(x_1) - \cos(x_1)\cdot x_1), x = 1.2..2.5, color = red, legend = \text{"tangent til f, i pkt. [x1,f(x1)], giver x2"}\right)$:

> $tangentx2 := plot\left((2\cdot\cos(x_2) - 1)\cdot x + 2\cdot(\sin(x_2) - \cos(x_2)\cdot x_2), x = 1.2..2.5, color = black, legend = \text{"tangent til f, i pkt. [x2,f(x2)], giver x3"}\right)$:

> $display(graf, punkter, tangentx0, tangentx1, tangentx2)$;



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